14 a) $y = x^2 - 5 \leftarrow f(x)$

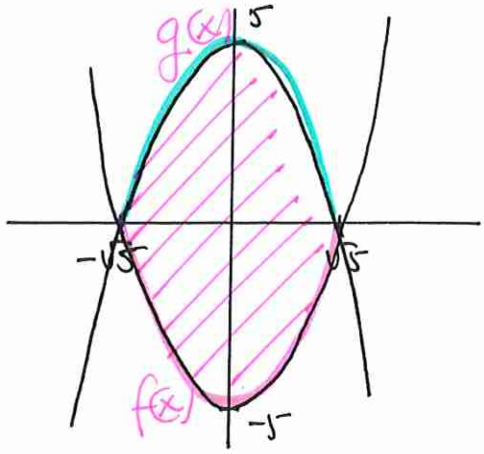
$y = -x^2 + 5 \leftarrow g(x)$

1) <u>Ebakidura-puntuak</u> $f(x) = g(x)$

$x^2 - 5 = -x^2 + 5$

$2x^2 = 10$

$x^2 = 5 \longrightarrow x = \pm\sqrt{5}$



<u>Irudikapena</u> (*) $f(x) = x^2 - 5$

Erpina $x = \frac{-b}{2a} = 0$ $y = -5$ $(0,-5)$

x ardatzarekin ebaketa puntuak

$0 = x^2 - 5$ $x = \pm\sqrt{5}$

(*)(*) $g(x) = -x^2 + 5$

Erpina $x = 0$ $y = 5$ $(0,5)$

x ardatzarekin ebaketa puntuak

$0 = -x^2 + 5 \rightarrow x = \pm\sqrt{5}$

2) <u>Konketa-funtzioa</u>

$g(x) - f(x) = (-x^2+5) - (x^2-5) = -2x^2 + 10.$

goikoa behekoa

3) <u>Azalera</u>

BARROW

$$A = \int_{-\sqrt{5}}^{\sqrt{5}} (-x^2+5) - (x^2-5)\, dx = \int_{-\sqrt{5}}^{\sqrt{5}} (-2x^2+10)\, dx =$$

$$= \left[-\frac{2x^3}{3} + 10x \right]_{-\sqrt{5}}^{\sqrt{5}} = \left(-\frac{2\sqrt{5}^3}{3} + 10\sqrt{5} \right) - \left(\frac{-2(-\sqrt{5})^3}{3} + 10(-\sqrt{5}) \right) =$$

$$= \left(-\frac{10\sqrt{5}}{3} + \frac{30\sqrt{5}}{3} \right) - \left(\frac{10\sqrt{5}}{3} - \frac{30\sqrt{5}}{3} \right) =$$

$$= \frac{20\sqrt{5}}{3} + \frac{20\sqrt{5}}{3} = \boxed{\frac{40\sqrt{5}}{3}\, u^2}$$

14 b) $y=x^2$ $\qquad f(x)=x^2$

$y^2=x \rightarrow y=\sqrt{x} \qquad g(x)=\sqrt{x}$

1.) EBAKIDURA PUNTUAK.

$f(x)=g(x)$

$x^2=\sqrt{x} \rightarrow x^4=x$

$x^4-x=0$

$x(x^3-1)=0$

$x_1=0 \quad x_2=1$

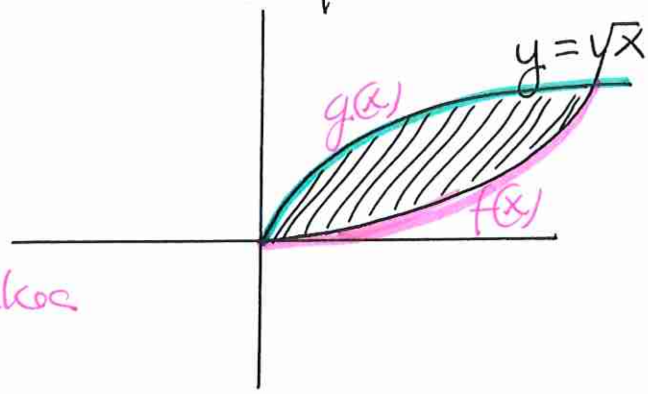
$(0,0) \quad (1,1)$

Irudikapena

$f(x)=x^2$ $\quad E(0,0)$ Ebak puntua ox ardatzapat $(0,0)$

$g(x)=\sqrt{x}$ Erroduna funtzioa

$Dom f=[0,+\infty)$



2.) Kenketa funtzioa goikoa-behekoa

$\underbrace{g(x)}-\underbrace{f(x)}=\sqrt{x}-x^2$

3.) Azalera

$$A=\int_0^1 (g(x)-f(x))\,dx=\int_0^1(\sqrt{x}-x^2)\,dx=$$

$$=\left[\frac{x^{1/2+1}}{1/2+1}-\frac{x^3}{3}\right]_0^1=\left[\frac{2}{3}\sqrt{x^3}-\frac{x^3}{3}\right]_0^1=$$

$$=\left(\frac{2}{3}\sqrt{1^3}-\frac{1}{3}\right)-\cancel{\left(\frac{2}{3}\sqrt{0^3}-\frac{0^3}{3}\right)}=\frac{2}{3}-\frac{1}{3}=\boxed{\frac{1}{3}u^2}$$

15 a) $y = 4 - x^2 \leftarrow f(x) = 4 - x^2$

$y = 8 - 2x^2 \leftarrow g(x) = 8 - 2x^2$

1) Ebakidura puntuak

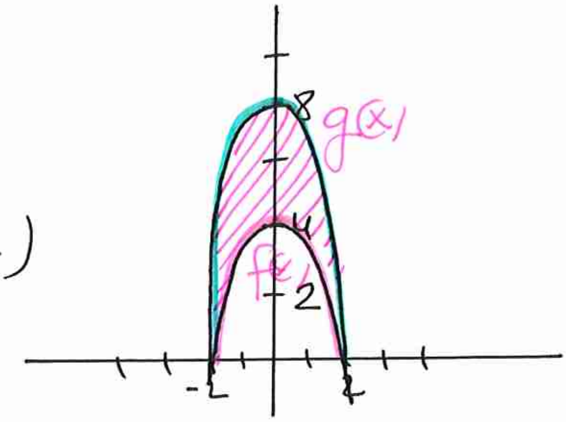
$f(x) = g(x)$ $\qquad x^2 - 4 = 0$ $\qquad x_1 = 2 \quad y_1 = 0$

$4 - x^2 = 8 - 2x^2$ $\qquad x = \pm 2$ $\qquad x_2 = -2 \quad y_2 = 0.$

Irudikapena

$\cap \quad y = 4 - x^2 \quad E(0,4)$ ox ardatiopot $0 = 4 - x^2 \rightarrow x = \pm 2$

$\cap \quad y = 8 - 2x^2 \quad E(0,8)$ ox ardatiopot $0 = 8 - 2x^2 \rightarrow x = \pm 2$



2) Kenketa funtzioa.

Goiko kurba - Beheko kurba

$\underbrace{g(x)} - \underbrace{f(x)} = (8 - 2x^2) - (4 - x^2)$

$= 4 - x^2$

3) Azalera - Barrow.

$$A = \int_{-2}^{2} (g(x) - f(x))\,dx = \int_{-2}^{2} (8 - 2x^2) - (4 - x^2)\,dx$$

$$= \int_{-2}^{2} (4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} =$$

$$= \left(4 \cdot 2 - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right) =$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 - \frac{-8}{3}\right) = 16 - \frac{16}{3} = \boxed{\frac{32}{3}\,u^2}$$

15 c) $\begin{cases} y = x^3 - 3x^2 + 3x \\ y = x \end{cases}$ $\quad f(x) = x^3 - 3x^2 + 3x$

$g(x) = x$

1.) EBAKIDURA PUNTUAK

$f(x) = g(x)$

$x^3 - 3x^2 + 3x = x$

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$$x = \frac{3 \pm \sqrt{3^2 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{2} = \begin{cases} \boxed{x_1 = 2} \\ \boxed{x_2 = 1} \end{cases}$$

$\boxed{x_3 = 0}$

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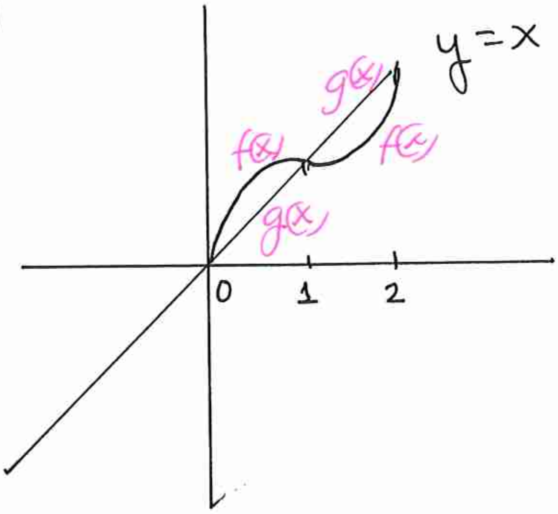
x	$f(x)$	$g(x)$
0,5	0,875 >	0,5
1.5	1,125 <	1,5



Azalera

$$A = \int_0^1 (f(x) - g(x))\,dx + \int_1^2 (g(x) - f(x))\,dx =$$

$$= \int_0^1 [(x^3 - 3x^2 + 3x) - x]\,dx + \int_1^2 [x - (x^3 - 3x^2 + 3x)]\,dx =$$

$$= \int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (-x^3 + 3x^2 - 2x)\,dx =$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[-\frac{x^4}{4} + x^3 - x^2\right]_1^2 =$$

$$\left(\frac{1}{4} - \cancel{1} + \cancel{1}\right) - (0) + \left(-\frac{16}{4} + 2^3 - 2^2\right) - \left(-\frac{1}{4} + \cancel{1} - \cancel{1}\right) =$$

$$\frac{1}{4} - \cancel{4} + \cancel{8} - \cancel{4} + \frac{1}{4} = \boxed{\frac{1}{2}\,u^2}$$